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ROUTING WITH STOCHASTIC DEMANDS AND SPLIT DELIVERIES

by

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ABSTRACT

This paper describes a heuristic algorithm for vehicle routing problems combining stochastic demands and split deliveries. The algorithm is tested on problems containing up to 130 demand points. For some demand parameters, allowing split deliveries in a stochastic context can yield substantial savings.

Key words : vehicle routing problem, stochastic demands, split deliveries

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RÉSUMÉ

On décrit dans cet article un algorithme heuristique pour des problèmes de tournées de véhicules avec demandes à la fois stochastiques et fractionnables. Des tests sont effectués sur de problèmes comportant jusqu'à 130 points de livraison. Pour certains paramètres de demande, on peut réaliser des économies substantielles en permettant des livraisons fractionnées dans un contexte stochastique.

Mots-clés : problèmes de tournées de véhicules, demandes stochastiques, livraisons fractionnées.

1. INTRODUCTION

The Vehicle Routing Problem (VRP) arises in a wide variety of distribution planning contexts. It can be described as follows. Consider a set $\{1,\ldots,n\}$ of cities or customers, where the first city is the depot. With every city $i>1$ is associated a non-negative demand. Let (c_{ij}) be a matrix of distances or travel costs between city pairs. There are at most m identical vehicles of capacity D based at the depot. The capacity constrained VRP consists of determining a set of m vehicle delivery routes of total minimum cost satisfying the following contraints:

(i) each route starts and ends at the depot;

(ii) every remaining city is visited exactly once by exactly one vehicle;

(iii) the total demand of any vehicle route does not exceed D.

In some contexts, customers have supplies instead of demands and vehicles then make collections. However, the mixed case is not considered in this paper.

There exists several other types of VRPs, the most common being the distance constrained VRP (Laporte et al., 1984) and the VRP with time windows (Desrochers et al., 1988). Comprehensive surveys can be found in Bodin et al., (1983), Christofides (1985) as well as Laporte and Nobert (1987). In recent years, some of the research effort has shifted from the classical VRP to more realistic variants of the problem. A number of interesting cases are reported in the book edited by Golden and Assad (1988). We are interested here in the VRP with stochastic demands and in the VRP with split deliveries.

Stochastic VRPs occur in contexts where some of the problem data are random variables; in our case we assume random demands. Typical examples of this class of problems are the delivery of home heating oil, or of fuel to

service-stations, and coin collection from telephone booths. Larson (1988) describes an application related to sludge disposal. In such contexts, vehicle routes may not be followed as planned since total demand of a planned route may turn out to exceed the vehicle capacity; this has direct consequences on problem definition and on solution methodology. For recent studies on stochastic VRPs, see Dror and Trudeau (1986), Laporte et al. (1989) and Dror et al. (1989). In the VRP with split deliveries, it is no longer assumed that every city is served by exactly one vehicle. Instead, its demand may be shared by several vehicles. Again, oil or fuel delivery is an area where split delivery routing makes sense. Dror and Trudeau (1990) have studied this case and have shown that allowing split deliveries may generate substantial savings.

As far as the authors are aware, no study has ever been made on VRPs combining stochastic demands and split deliveries. This a particularly important and realistic combination for the following reason. In a stochastic context, route failures are common. These occur when a vehicle becomes empty in the process of fulfilling a customer's demand. In such a case, several recourse actions are possible, such as returning to the depot for replenishment and resuming deliveries at the point of failure. Such a policy may of course be suboptimal and it could be preferable for the vehicle involved to move to a different customer and to let another vehicle serve the customer whose delivery could not be completed. In other words, split deliveries could prove a natural and economical answer to route failures.

The object of this paper is to describe a heuristic algorithm for VRPs with stochastic demands and split deliveries. It combines into a single procedure two heuristics developed independently for each of the two variants. Extensive computational results confirm the efficiency and the robustness of the proposed approach.

2. ALGORITHM

The algorithm alternates between two main routines, STOCH and SPLIT, that solve a VRP with stochastic demands and a VRP with split deliveries, respectively. The transition from STOCH to SPLIT is quite straightforward. Going from SPLIT to STOCH requires some adjustments. Before explaining the global algorithm, we first describe STOCH and SPLIT.

2.1 Vehicle routing with stochastic demands (STOCH)

Let $\mathcal{F}_i$ be a discrete or continuous non-negative independent random variable denoting the demand of customer i and, for the sake of simplicity, assume $\mathcal{F}_i < D$ for all i. If this not the case, artificial customers can be created by appropriately splitting the demands of customers for which $\mathcal{F}_i > D$. Each $\mathcal{F}_i$ has a finite mean μ_i and a finite variance σ_i^2 and we assume the variables $(\mathcal{F}_i - \mu_i)/\sigma_i$ to be identically distributed. The problem consists of determining a set of m planned vehicle routes. Each route is followed by a vehicle until the sum of demands of all visited customers exceeds D. At this point, failure is said to occur and the following recourse policy is applied. All remaining customers on the route whose demand has not yet been satisfied are visited by means of return trips to and from the depot. This policy penalizes heavily the occurence of failure on a planned route. Under these rules and assumptions, the probability of failure at any customer along a planned route can easily be computed - see Dror and Trudeau (1986), for example - and the expected cost of any route (including the planned cost and the cost of recourse action) can therefore be derived. The objective is to determine a set of vehicle routes of least total expected costs. In order to solve this problem, Dror and Trudeau (1986) have suggested an adaptation of the Clarke and Wright (1964) algorithm. In the original algorithm, the decision whether to combine two routes with

customers i and j respectively into a single route depends on the value of the savings $s_{ij} = c_{i1} + c_{1j} - c_{ij}$ and on the feasibility of such an operation. In STOCH, the savings term is replaced by $\max \{c'_i + c'_j - c'_{ij}, c'_i + c'_j - c'_{ji}\}$, where c'_i is the expected cost of the current route including i, and c'_{ij} is the expected cost of the combined route where i immediately precedes j.

2.2 Vehicle routing with split deliveries (SPLIT)

The SPLIT algorithm starts from a feasible VRP solution (obtained by the Clarke and Wright algorithm, for example) and gradually improves it by applying a sequence of routines linked by conditional statements. Since the SPLIT algorithm has been fully described in Dror and Trudeau (1990), we only provide a summary:

Step 1. Improve the initial solution by means of the node interchange procedure described in Dror and Levy (1986).

Step 2. Apply a 2-Opt procedure (Lin, 1965) after each route construction.

Step 3. Attempt to obtain some improvements by splitting certain demand points between a number of vehicle routes.

Step 4. Seek further improvements by introducing new routes; Dror and Trudeau (1990) show that this may indeed be profitable when a demand point is split between several routes.

2.3 The global algorithm

The global algorithm can now be described

Step 1. Set $z^* := \infty$.

Step 2. Apply the STOCH procedure. Let z be the cost of the optimal solution. If $z > z^*$, stop.

Step 3. Apply the SPLIT procedure assuming mean demands. If any demand ξ_i is split between k vehicles in proportions $p_1, p_2, \ldots, p_k$, replace customer i with k customers having demands ξ_i^k of means $p_k \mu_i$ and of variances $p_k \eta_i^2$. Set the distance between any two replies of the same splitted customer equal to infinity. Compute the optimal solution cost z. If $z < z^*$, set $z^* := z$. Return to Step 2. ∎

This description requires an observation. After Step 2, it is necessary to set to infinity the distance between split customers for otherwise, these would immediately be lumped back together in the next execution of Step 1 and hence, the algorithm would always terminate after one pass. However, distances between split customers can safely be set back to zero after the first phase of the Clarke and Wright algorithm in Step 1. In other words, while split fractions of a customers are initially assigned to different vehicles, some parts are allowed to be subsequently reunited.

3. COMPUTATIONAL RESULTS

The algorithm was tested on 900 problems generated as follows. Ten sets of coordinates were generated for each of the three sizes n=75, 115, and 130. The first 75-city problem is described in Eilon et al. (1971). The remaining 75-city problems, and all 115 and 130-city problems were obtained by randomly generating coordinates in $[0,80]^2$ according to a uniform distribution. The vehicle capacity was set at 160 in all cases. For each size and for each set of coordinates, thirty problems were obtained by generating six sets of mean demands μ_i, according to a uniform distribution in $[160\alpha, 160\beta]$, for each of the five cases $[\alpha,\beta]$=[.1,.3,], [.1,.5], [.1,.9], [.3,.5] and [.3,.7]. The value of σ_i was set equal to $(T\mu_i/a)^b$, where T was randomly generated in [0,1] according to a uniform distribution, and the values of a and b were chosen as follows: a=3, b=2 for $[\alpha,\beta]$=[.1,.3,], [.1,.5], [.3,.5]; a=8, b=1.2 for $[\alpha,\beta]$=[.3,.7]; a=16, b=1 for $[\alpha,\beta]$=[.1,.9]. The STOCH subroutine was written in FORTRAN, while SPLIT was written in PASCAL. All programs were run on a CDC CYBER 855 computer, but all times reported are CYBER 173 equivalent. Computational results are summarized in Table 1.

The various column heading can be interpreted as follows:

n : number of cities;

$[\alpha,\beta]$: control parameters for demand generation;

ITR : number of iterations, i.e. number of executions of the SPLIT subroutine;

COMPUTATION TIME : total computation time in seconds in STOCH and in SPLIT.

COST IMPROVEMENT : value of $(z_2-z_1)/z_1$ where z_1= solution value after the first execution of STOCH, and z_2= final solution value;

- AVERAGE : mean value for all 60 problems with a given n and a given $[\alpha,\beta]$;
- MAXIMUM : largest value over 60 problems;
- FREQUENCY: percentage of problems for which a positive improvement was observed

ROUTE IMPROVEMENT: value of $(m_2- m_1)/m_1$ where m_1= number of vehicle routes after the first execution of STOCH, and m_2= number of vehicle routes in the final solution. The three columns AVERAGE, MAXIMUM and FREQUENCY are interpreted as above.

The columns ITR, STOCH, SPLIT and AVERAGE represent mean values over 60 problems.

Our results indicate that on the average, the algorithm terminated after a limited number of iterations, between 1 and 3. Although the STOCH routine was always executed one more time than SPLIT, the total time spent in SPLIT is generally higher than the time spent in STOCH. In fact, the relative time spent in SPLIT tends to increase with n. In general, large values of n tend to produce larger gains and as problem size becomes larger, the percentage of problems for which a gain is observed tends to increase and sometimes exceeds 85%. Although our algorithm makes no

Table 1. Summary of computational results

n	$[\alpha,\beta]$	ITR	COMPUTATION TIME		COST IMPROVEMENT			ROUTE IMPROVEMENT		
			STOCH	SPLIT	AVERAGE	MAXIMUM	FREQUENCY	AVERAGE	MAXIMUM	FREQUENCY
75	[.1,.3]	1.37	132.48	117.47	0.1%	2%	28.3%	0.3%	5.9%	5.0%
	[.1,.5]	1.77	153.40	193.38	0.5%	3%	53.3%	2.7%	10.7%	43.3%
	[.1,.9]	1.07	95.83	138.46	0.1%	2%	8.3%	0.0%	10.4%	5.0%
	[.3,.5]	1.63	133.16	163.21	0.3%	2%	46.7%	3.4%	10.8%	45.0%
	[.3,.7]	1.80	138.55	236.85	0.8%	3%	56.7%	7.7%	20.0%	58.3%
115	[.1,.3]	1.62	350.74	576.69	0.2%	2%	45.0%	1.2%	7.1%	28.3%
	[.1,.5]	2.25	449.97	1049.37	0.7%	2%	80.0%	3.7%	9.5%	73.3%
	[.1,.9]	1.08	249.59	305.06	0.0%	1%	6.6%	0.8%	11.7%	8.3%
	[.3,.5]	1.73	364.37	727.67	0.3%	2%	60.0%	3.7%	10.7%	60.0%
	[.3,.7]	2.17	428.15	967.12	1.6%	4%	83.3%	11.8%	24.7%	83.3%
130	[.1,.3]	1.80	479.55	938.04	0.3%	2%	50.0%	1.1%	9.4%	28.3%
	[.1,.5]	2.58	642.30	1861.37	1.0%	3%	83.3%	4.3%	11.8%	81.7%
	[.1,.9]	1.25	362.34	577.35	0.3%	3%	25.0%	2.9%	16.1%	25.0%
	[.3,.5]	1.80	481.04	1160.16	0.5%	2%	73.3%	4.3%	9.4%	73.3%
	[.3,.7]	2.08	543.20	1245.75	3.4%	5%	86.6%	11.5%	19.5%	86.6%

specific attempt at reducing the number of vehicle routes, route savings (i.e. vehicle savings) are generated as a by-product of the overall cost reductions. This is particularly true for $[\alpha,\beta]=[.3,.5]$ and $[.3,.7]$. In the latter case, route savings exceed 11% and can be as high as 24%. There can therefore be a distinct advantage in running the combined STOCH-SPLIT algorithm, rather than STOCH alone, especially since the overall computing time is always very reasonable. In their paper on split delivery routing in a deterministic context, Dror and Trudeau (1990) reach a similar conclusion, albeit stronger.

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REFERENCES

Bodin, L.D., B.L. Golden, A.A. Assad and M.O. Ball (1983), Routing and Scheduling of Vehicles and Crew. The State of the Art. Computers and Operations Research 10, 67-211.

Christofides, N. (1985), Vehicle Routing, in The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization, pp. 431-448, E.L. Lawler, J.K. Lenstra, A.H.G. Rinnooy Kan and D.B. Schmoys (eds.), Wiley, Chichester.

Clarke, C. and J.W. Wright (1964), Scheduling of Vehicles from a Central Depot to a Number of Delivery Points, Operations Research 12, 568-581.

Desrochers, M., J.K. Lenstra, M.W.P. Savelsbergh and F. Soumis (1988), Vehicle Routing with Time Windows: Optimization and Approximation, in Vehicle Routing: Methods and Studies, pp. 65-84, B.L. Golden and A.A. Assad (eds), North-Holland, Amsterdam.

Dror, M., G. Laporte and P. Trudeau (1989), Vehicle Routing with Stochastic Demands: Properties and Solution Frameworks, Transportation Science 23, 166-176.

Dror, M. and L. Levy (1986), A Vehicle Routing Improvement Algorithm: Comparison of "Greedy" and a "Matching" Implementation for Inventory Routing, Computers & Operations Research 13, 33-45.

Dror, M. and P. Trudeau (1986), Stochastic Vehicle Routing with Modified Savings Algorithm", European Journal of Operational Research 23, 228-235.

Dror, M. and P. Trudeau (1990), Split Delivery Routing, Naval Research Logistics 37, 383-402.

Eilon, S., C.D.T. Watson-Gandy and N. Christ fides (1971), Distribution Management: Mathematical Modelling and Practical Analysis, Griffin, London.

Golden, B.L. and A.A. Assad (1988), Vehicle Routing: Methods and Studies, North-Holland, Amsterdam.

Laporte, G., M. Desrochers and Y. Nobert (1984), Two Exact Algorithms for the Distance Constrained Vehicle Routing Problem, Networks 14, 161-172.

Laporte, G., F. Louveaux and H. Mercure (1989), Models and Exact Solutions for a Class of Stochastic Location-Routing Problems, European Journal of Operational Research 39,71-78.

Laporte, G. and Y. Nobert (1987), Exact Algorithms for the Vehicle Routing Problem, in Surveys in Combinatorial Optimization, pp. 147-184, S. Martello, G. Laporte, M. Minoux and C. Ribeiro (eds.), North-Holland, Amsterdam.

Larson, R.C. (1988), Transportation of Sludge to the 106 Mile Site: An Inventory Routing Algorithm for Fleet Sizing and Logistic System Design, Transportation Science 22, 186-198.

Lin, S. (1965), Computer Solutions of the Traveling Salesman Problem, Bell System Technical Journal 44, 2245-2269.